



Safety Stock Determination of Uncertain Demand and Mutually Dependent Variables

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ABSTRACT

Safety stock is that point, where the user finds a comfort zone between overstock and understock situation. It is defined as the buffer inventory have to be kept to deal with differences between supply and demand. There are different variables to be considered while determining safety stock. In this writing there is an effort to establish a model that include direct and indirect cost related to inventory. The inclusion of Ordering cost, holding cost, Product price, Time, Demand, Demand Variation, Lead time, Mean lead time, Errors in Forecasting, Deviation of lead time etc. are used in this model. This model works with economic order quantity, regression, and forecasting error calculation to estimate safety stock while reducing human judgment error in the calculation.

Keywords: Dependent Variables, Direct and Indirect Inventory Cost, Safety Stock, Uncertain Demand.

JEL Codes: C20, D21, J29.

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1. Introduction

Inventory is always a big concern for any organization. Whether it is overstock or understock, which occurs often times, becomes a headache for managers. Therefore Inventory is regarded as liability in Supply chain Arena, as it bears a significant business cost (Wang & Croxton, 2010). In 2008, firms operating in the United States spent \$420 billion to maintain inventories valued at almost \$2 trillion (Wilson 2009). In a survey of 190 producers, distributors and retailers, most of the respondents reported that improving their service level was the primary goal of their inventory management (Kanet, Gorman & Sto, 2010). In addition to that, stocking sufficient inventories, low storage costs which are usually influenced by stock rates – have to be ensured (Hon, 2005). To retain the balance, Safety-stock has to be maintained in order to satisfy requirements even when parameters such as the demand, replenishment times and replenishment quantities are volatile (Ruiz & Mahmoodi, 2010).

Safety stock is that point, where the user finds a comfort zone between overstock and understock situation. It is defined as inventory that absorbs various differences between supply and demand (Yamazaki, Shida, & Kanazawa, 2015). Traditionally, safety stock is Safety inventory protects against inventory uncertainty by ensuring there is enough product available to maintain desired service

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levels. (Haddley, 2004). According to Haddley, Three main elements give rise to the differences between planned inventory and actual inventory levels:

- demand deviations,
- supply deviations,
- inventory accuracy (2004)

These differences result from

- Changes in the timing of customer purchases,
- differences between average demand and actual demand,
- fluctuations in demand, machine breakdowns,
- employee absenteeism,
- material shortages,
- Product defects, etc. (Yamazaki, Shida, & Kanazawa, 2015).
- Late delivery,
- Short shipments,
- Production delays,
- Production yield differing from planned, and
- Substandard materials and production. (Haddley, 2004)

And of course there could be many reasons causing inventory variability including but not limited to Procurement planning, Production and logistics activities etc.

Objective: The objective of the paper is to study the variables considered in the existing literature to determine safety stock and look for other areas that could be considered. Also, look for more comprehensive way to derive safety stock.

Research question: Safety stock is directly related to Demand of products. In an uncertain scenario, organizations has to forecast the demand. Besides, after forecasting, Organizations has to divide the forecasted data into acceptable quantity of orders that is also economical according to the organization resources. Economic order quantity (EOQ) is the one of the acceptable solutions for that. Therefore, we would like to include this methods into safety stock calculations and understand the applications.

And so, our question is: Are inclusions of (a) Forecasted Demand (b) forecasting error and (c) EOQ data could help to determine Safety stock more inclusively and accurately?

Implications: This paper will try to come up with a comprehensive safety stock model that could be used in uncertain demand scenario determination when mutually dependent variables are considered.

2. Literature review

Inventory control models in the research literature and in textbooks typically assume that the demand distribution and all its parameters are known. However, in practice such information is not available, and future demands have to be forecasted based on historical observations (Prak, Teunter, & Syntetos, 2017). No forecasting procedure produces perfect forecasts (Prak, Teunter, & Syntetos, 2017). For unbiased forecasting procedures, not only methodical forecasting but also continuous forecast errors calculation of demand is necessary.

There are many forecast method found in the literature. Surprisingly, some inventory textbooks do not mention forecasting at all (Muckstadt & Sapra, 2010; Zipkin, 2000) and also they do consider to discuss the use of forecasts and errors consideration in the decision models (Waters, 2012). Some inventory textbooks, such as Silver, Pyke, and Peterson (1998) and Axsäter (2016), do indicate that parameters have to be estimated in practical applications, and that the resulting forecast error should be taken into account. Some books considers per-period demand forecast errors (e.g. Estimated via Mean Squared Error, MSE, or Mean Absolute Deviation, MAD).

The lead time forecast errors can correlated and has been pointed out by some researchers. Silver et al. (1998) noted that the relationship between the forecast error during the lead time and that during the forecast interval. Beutel and Minner (2012) discuss a framework of least squares demand forecasting and inventory decision making.

A common approach to inventory decision making under un-known demand parameters is the Bayesian approach, which was initiated by Dvoretzky, Kiefer, and Wolfowitz (1952), Scarf (1959), Karlin (1960), and Iglehart (1964). Applications of Bayesian estimation in dynamic programming efforts are

given by e.g. [Azoury \(1985\)](#) and [Lariviere and Porteus \(1999\)](#). It has been found by them that specific, optimal order quantities are numerically tough to derive. This led to the development of heuristics, that essentially reduce multi-period problems to single-period problems, and typically assume a negligible lead time ([Chen, 2010](#)). Numerical Bayesian studies mainly involve single-parameter demand distributions, or, if the distribution has more than one parameter, impose that one or more parameters are known. Besides, [Rajashree Kamath and Pakkala \(2002\)](#) use a lognormal distribution with a known variance. This restriction is theoretically convenient, because it ensures that the posterior demand distribution is again (log) normal, but it is practically incorrect, as the variance has to be estimated as well.

A different approach is tried by [Hayes \(1969\)](#), who introduces the method of minimizing the Expected Total Operating Cost, which follows from the sampling distribution of the parameter estimators. [Janssen, Strijbosch, and Brekelmans \(2009\)](#) seek the optimal 'upward biases' in the traditional stock level calculation via simulation. Similarly, [Strijbosch and Moors \(2005\)](#) consider a batch ordering (r, Q) policy under normally distributed demand with unknown parameters and zero lead time.

[Lippman and McCardle \(1997\)](#) examined the relationship between equilibrium inventory levels and the allocation rule. Along with them [Netessine and Rudi \(2003\)](#) established the uniqueness of the equilibrium for the n -product case, and extended the work of [Parlar \(1988\)](#). [Avsar and Gursoy \(2002\)](#) treated the same condition on an infinite horizon. [Ahn and Olsen \(2007\)](#) worked on multiple periods with consideration of a subscription sale. [Netessine, Rudi, and Wang \(2006\)](#) presented four scenarios of customers' backlogging and transfer behaviour in a dynamic environment.

[Khmelnitsky and Gerchak \(2002\)](#) analysed a continuous review policy by using a deterministic inventory model in which demand is a function of the instantaneous inventory level, where shortages are possible and production capacity is limited. [Baker and Urban \(1988\)](#) investigated the continuous, deterministic case of an inventory system in which the demand rate for an item is a function of that inventory level. [Kelle and Silver \(1989\)](#) identified a reorder point for purchasing new containers in which products are sold. [Yuan and Cheung \(1998\)](#) developed for this model a continuous review (s, S) policy with returns based on the sum of the on-hand stock and the number of items in the field.

[Decroix \(2006\)](#) and [Neuts \(1964\)](#) proposed an optimal order-up-to policy with regular and emergency replenishments for the case in which the regular order lead time is one period and the emergency lead time is zero. [Whittemore and Saunders \(1977\)](#) derived the optimal policy for the case in which the regular and replenishment lead times are multiples of the periodic review period. [Rosenshine and Obee \(1976\)](#) investigated a standing order inventory system in which a constant quantity is received every period, and a fixed-size emergency order can be placed when the inventory falls below the reordering point.

[Blumenfeld, Hall, and Jordan \(1985\)](#) analysed the trade-off between expediting cost and safety stock cost. Emergency orders are considered to be sufficiently large to avoid stock outs and to be received with a zero lead time. [Chiang and Gutierrez \(1996\)](#) analysed inventory models in which the lead time is shorter than the periodic review period with a periodic review system. [Teunter and Vlachos \(2001\)](#) allowed more than one emergency order per review period. [Alfredsson and Verrijdt \(1999\)](#) showed the effectiveness of emergency flexibility, including transshipments from other retailers. [Chartniyom et al. \(2007\)](#) proposed a solution approach to avoiding stock outs that incorporates two options, lateral transshipment and emergency order.

[Weiss \(1980\)](#) proposed an optimal reorder point and order-up-to level with continuous review and a zero lead time. [Kalpakam and Arivarignan \(1988\)](#) investigated a model in which there is an exponential lifetime and a zero reorder point. [Nahmias, Perry, and Stadje \(2004\)](#) considered the condition in which the demand rate is not affected by the product lifetime. [Avinadav and Arponen \(2009\)](#) assumed that the demand rate increases in proportion to lifetime. [Avinadav, Herbon, and Spiegel \(2013\)](#) modelled demand that decreases in proportion to price and polynomially with lifetime.

There are other streams available in literatures. But in terms of safety stock, most of them considered it as inclusive effort while working with inventory as a whole. Moreover, an optimal policy has been proposed for considering the production system, the order sizes, the planning horizon etc. Considering all these, the authors some time focus on safety stock more or less. But still no one skips the specification of safety stock one way or another. However, they have common effort, making the models complex and realistic.

2.1 Safety stock calculations

However, Specifically in terms of safety stock, there are many safety stock calculating formulas. The following Equations are widely used (e.g., [Chatfield et al. 2004](#), [Chopra et al. 2004](#), and [Ray 1981](#)).

Safety stock (SS) is calculated using:

$$SS = k \sigma z \quad (\text{e.g., } \text{Chatfield et al. 2004}, \text{ Chopra et al. 2004}, \text{ and Ray 1981}).$$

Where k is a safety factor parameter based on the distribution of the total demand in a lead time and σz is the standard deviation of total demand in a lead time (Z). The demand in a lead time is a random sum (lead time) of random numbers (demand). Therefore, probability theory informs us that, when lead time and demand are uncorrelated, the standard deviation of total demand in a lead time is ([Feller 1957](#))

$$\sigma_z = \sqrt{\mu_t \sigma_d^2 + \mu_d^2 \sigma_t^2}$$

([Chopra and Meindl 2007](#))

Where,

μ_t = mean lead time

μ_d = mean demand

σ_t = standard deviation of time

σ_d = standard deviation of demand

Different textbooks, Inventory reviews and software are also following these rules ([Atkinson 2005](#), [Chopra and Meindl 2007](#), [Neale et al. 2003](#), [Silver et al. 1998](#))

[Yamazaki, Shida, & Kanazawa \(2015\)](#) define Safety stock is defined as inventory that absorbs various differences between supply and demand. These differences result from changes in the timing of customer purchases, differences between average demand and actual demand, fluctuations in demand, machine breakdowns, employee absenteeism, material shortages, product defects, etc. Traditionally, safety stock is given by

$$sS = z * \sigma_t * \sqrt{(T + L)}$$

z : Safety Factor.

σ_t : Standard deviation of demand per unit time

T = Review period.

L = Lead Time

([Yamazaki, Shida, & Kanazawa, 2015](#))

Another model is proposed by [scheidler & Workman \(2009\)](#), they proposed easier method to work with safety inventory. The equation is:

$$\text{Safety Stock} = SS_{\text{Demand}} + SS_{\text{Supply}} + SS_{\text{Strategic}}$$

$$SS_{\text{Demand}} = (1 - \text{Forecast Accuracy}) \times \text{Lead Time}$$

$$= \text{Days of Coverage Needed}$$

$$SS_{\text{Supply}} = (1 - \text{Supplier on time delivery}) \times \text{Lead Time}$$

$$= \text{Days of Coverage Needed}$$

$$SS_{\text{Strategic}} = \text{Days of Coverage Needed}$$

([scheidler, Workman, 2009](#))

[Eppen & Martin \(1988\)](#), proposed another method but close to [Feller \(1957\)](#). The core difference is [Feller \(1957\)](#) proposed root over the of multiplication addition of standard deviation and mean lead time and demand. Where [Eppen & Martin \(1988\)](#) did not use the root instead used multiple variables to compute safety stock

We use the following notation throughout.

Let

W = lead time demand, a continuous random variable,

X = demand during one time period, a continuous random variable,

Y = length of lead time in time periods, a discrete random variable,

w = lead time demand given period lead time,

μ_x = mean demand during one time period.

μ_y = mean lead time,

σ_x = demand variance during one time period,

σ_y = lead time variance,

(Feller 1960)

$$\mu_w = \mu_x + \mu_y$$

$$\sigma_w^2 = (\sigma_x^2)(\mu_y) + (\sigma_y^2)(\mu_x)^2$$

(Eppen & Martin, 1988).

In inventory literatures, there are many complex theories used, but in most of the cases, the users of the theories have hard time to understand and use the models. One of the main reasons is the inventory managers are busy of their daily operations. In a store or warehouse, there are multiple products. Most of the time it is not possible for them to use any complex formula to derive safety stock levels. Therefore there is need of using close to complex but simple but inclusion of different variable data used formula that will be beneficial for root level users.

In terms of Safety stock calculation, most of the time literatures use lead time and demand variation. But demand variation do no stay always same. Besides, Demand is not fixed most of the time. Therefore demand is uncertain. But we see little effort in demand forecasting efforts in the literature. Some (Muckstadt & Sapra, 2010; Zipkin, 2000) authors considered demand errors seriously. But still comprehensive demand forecasting and error methods are not used in terms of safety stock calculations. As most of the calculations deal with certain data (Silver et al. 1998). Here our effort is to deal with uncertain data. In that case inclusion of demand forecasting is necessary.

Besides, Demand forecasting Economic order quantity (Roach, 2005) could also be used to derive safety stock calculation. As EOQ is already recognized for assessing optimum safety stock, we could use this in our safety stock calculation. Although EOQ has some drawbacks (Roach, 2005), But still it is one of the most recognized formula to derive ultimate inventory. Besides, we will not use EOQ alone. Inclusion on other variables in an equation will reduce the error of the EOQ in our calculation.

3. Proposed model: Demand forecasting in uncertain scenario

In inventory demand, most of the cases data are interdependent. To deal with the mutually depended data, we work with linier or multi-linier regression. In our case linier regression will be used. If more variables are there multi-linier regression will be useful.

The regression formula is following:

$$b = \frac{\sum YX - n\bar{Y}\bar{X}}{\sum X^2 - n\bar{X}^2}$$

$$a = \bar{D} - b\bar{X}$$

$$D(F) = a + bX$$

(Mark Lunt, 2015)

1st step: Demand forecast using linier regression

In our case, Lead time and Demand are two interdependent factors. In our test case, we will consider following data for our study. It is hypothetical data.

SL.	Lead time (X)	Demand (Y)	(F)Forecast
1	8	1200	?
2	9	1000	?
3	8	1500	?
4	10	1300	?
5	12	1100	?
6	9	1000	?
7	12	?	← ?

In this case, lead time and demand are mutually dependent variables. We have data of 6 period. Lead time for 1st month is 8 days and demand was 1200. Same goes on with subsequent months. We will calculate the regression forecast of each period. (Schneider, Hommel, & Blettner, 2010). In this way forecast of 7th period will derive, assuming the lead time is known, demand is unknown. In this way we will find the forecast of 7th period ans will regard as Forecasted Demand of period 7 (Ballou, 2004).

2nd step: Find out forecast error

After bringing the forecasted data, now it is time to work with forecast error. Generally, Error is the differences between the actual and forecast values, in equation, it is defined:

$$e_t = y_t - \hat{y}_t$$

(Cuhadar, Cogurcu, & Kukrer, 2014)

However, We are working with popular method of forecasting Mean Absolute error (MAE), Sum of Squared Error (SSE), Mean squared Error (MSE), Square root of MSE (RMSE) (Hyndman & Athanasopoulos, 2016).

Forecasts and error measures for a simple regression model

	Lead time	Demand	FCST	ERROR	ABS ERR	SQ ERR
						2087.41
	8	1200	1245.69	-45.69	45.688	9
					209.31	43812.5
	9	1000	1209.32	-209.32	5	85
						64674.4
	8	1500	1245.69	254.31	254.312	53
					127.05	16144.02
	10	1300	1172.94	127.06	9	9
	12	1100	1100.19	-0.19	0.193	0.037
					209.31	43812.5
	9	1000	1209.32	-209.32	5	85
	12		1100.19			
Number of data points (n)	7	6				
Mean (AVERAGE)	9.71	1183.33	1183.33	-13.86	140.98	28421.85
Sample standard deviation (STDEV)	1.70	194.08	61.99	184.05		
Sample variance (VAR)	2.91	37666.74	3843.14	33875.81		
Correlation of X & Y (r = CORREL(X,Y))	-0.32				Mean absolute error (MAE)	140.98
Regression slope coefficient (b = r*STDEV(Y)/STDEV(X))	-36.37				Sum of Squared Errors (SSE)	170531.1
Regression intercept (a = AVERAGE(Y)-b*AVERAGE(X))	1536.68				MSE (SSE/(n-p))	42632.7
Square of correlation coefficient (r squared)	0.10203				RMSE (square root of MSE)	206.48
Unadjusted R-squared (1-VAR(ERR)/VAR(Y))	0.100642				Critical t-value (95%, n-p d.f.)	2.77
Adjusted R-squared (1-MSE/VAR(Y))	-0.13184					

*Calculation A: Demand forecasting & Error calculation. Step 1 & 2. (Schneider, Hommel, G & Blettner, 2010; Hyndman, & Athanasopoulos, 2016; Chai, and Draxler, 2014; Mark Lunt, 2015; Swift, & Piff, 2014; Barde, & Barde, 2012).

3rd step: Deriving EOQ**EOQ:**

The Economic Order Quantity (EOQ) formula is one of the most pervasive formulas in the business and engineering literature. It is the level of inventory that minimizes total inventory holding costs and ordering costs (Ngugi, Aiyabei, Maroko & Ngugi, 2012). Engineers study the EOQ formula in engineering economy classes and in industrial engineering classes (Roach, 2005). Business students study the EOQ formula in both finance and operations management classes. Sometimes the EOQ formula is studied for practical reasons as part of specific applications in inventory systems (Roach, 2005).

EOQ decision rules

1 These rules indicate what quantities could be ordered, so as to balance optimum the cost associated with them. Balancing the fixed costs per lot against the carrying costs is the basis for arriving at the economic order quantity (Roach, 2005). The development of the formula used the following assumptions:

- Demand is continuous and constant (Le. rate of depletion of inventory is constant); (Harris 1913),
- The process continues infinitely; and
- There are no quantity constraints (on order quantity or storage capacity). Initially, the development also assumes the following (later these assumptions would be relaxed to develop variants of the basic rule):
- Replenishment is instantaneous;
- No shortages are allowed; and
- Costs are time and quantity invariant. (Roach, 2005).

The development uses the following is the notation:

Q = order quantity

h = inventory holding cost

K= order cost in dollars per order;

D = demand rate pieces per unit time (year); and

The rule is

$$EOQ = \frac{\sqrt{2 KD}}{h}$$

(Schwarz, 2007; Harris 1913),

For our case

Demand=Forecasted Demand of period 7= received from Calculation above.

Cost of Order= we assume in this case 1000

Cost of the product: we assume=1100

Carrying cost= we assume =20% of Purchase price.

$$EOQ = \frac{\sqrt{2 KD}}{h} = EOQ = \frac{\sqrt{2 \times 1100 \times 1000}}{220} = 100$$

*Calculation B: EOQ calculation

4th step: Set review period

In this step, we have to set a review period. The review period is supposed to be set by the organization. As it depends on the policy, practice, supplier and capacity of organization.

In our case we are assuming, review period= 10 Days

5th step: Get mean absolute error

In this step, we will get mean absolute error to find the errors among actual demand and forecasting of different period. These will help to reduce the errors likely to happen in our forecasted 7th period.

$$MAPE = \frac{\sum_{i=1}^n |y_i - x_i|}{n}$$

(Chai & Draxler, 2014)

6th step: Derive mean lead time

Lead time is another important factors in demand forecasting. In this step we are trying to get the central tendency or mean of lead time so far occurred during supply of a particular product. Lead time means the time took to receive the product after ordering in a specific time.

Mean lead time, $\mu_t = \sum X/n$

7th step: Derive standard deviation of Lead time

We use standard deviation of lead time to from lead time history so far and calculate the deviation of lead time happed so far. This also reduces the errors in lead time we are considering in our equation. In our case, 7th period lead time 12 days assumption error is minimized the standard deviation of lead time calculation.

Standard deviation of lead time, $\sigma_{LT} = \frac{\sum_{i=1}^N (x_i - \bar{x})^2}{N-1}$
(Barde & Barde, 2012; Swift & Piff, 2014).

8th Step: Calculating all data altogether

The proposed model is mention below. The rational of this model is discussed later part of this writing.

Safety Stock = $\frac{EOQ^*}{Review_Period} + \text{mean absolute error of demand} \times \text{mean lead time} \times \text{Standard deviation of lead time}$

* EOQ after regression demand forecasted and error calculated demand

We can write this model, in short,

$$Safety\ Stock = \frac{EOQ^*}{R} + MAE \times \mu t \times \sigma_{LT}$$

In our case, the calculation is:

$$= \frac{100}{10} + 140 \times 9.71 \times 1.70 = 2476.05$$

*Calculation C: Safety stock calculation

4. Rational of the calculation

In our calculation, there are some reasons behind inclusion of all the formulas in Safety stock determination. We got the Demand from regression analysis as the demand was unknown for desired period. Besides Lead time and Demand are mutually dependent. Using the demand we calculate EOQ to get the specific amount to order each time by optimizing the inventory related costs. After that EOQ is divided by the review interval to get per day demand from EOQ, indicate optimum demand each day we need. Now, this amount is added with mean absolute error. As the error indicates missing amount in demand, could be positive or negative, it is added with in our calculation. Then the derived data is multiplied by mean lead time to get the demand of the lead time period. As lead time could vary, as we see it varied before in our case, we need to multiply the derived data with the standard deviation of lead time to reduce the error of lead time took so far before.

In our case, the result is 2476, which almost 2 times of our mean demand (1185). By looking at data, it supposed to acceptable safety stock as it indicates safe inventory amount for 2 periods considering the lead time. If unfortunately one time delivery fails, the safety stock is enough to support the interim period (1 failed delivery period+1 New order receive lead time period=2 period of inventory). Besides, Considering the Review period on 10 days (1 period) and average lead time of 10 days adds 20 days that is equals to 2 period of demand. Our Safety stock indicates this 2 period demand (20 days). Besides, as the demand data was derived from extensive analysis of Linier regression, Error calculation and EOQ analysis, we can expect the demand data contains less error in derived safety stock data.

Other calculations in the 'Calculation 1' chart. For example, Sample variance, Correlation, Square of correlation coefficient etc. was to calculate the validity of the data used in our model.

5. Application

This safety stock model could be used in uncertain demand scenario determination when mutually dependent variables are considered. Besides, in other cases the model could be used in a trial basis to see the application.

Here, the point to be noted that the, the model is still in theoretical format. It needs to be used in real life scenario.

6. Conclusion

This is a comprehensive model that considers different factors in safety stock determination both direct and indirect cost related. There is inclusion of Ordering cost, holding cost, Product price, Time, Demand, Demand Variation, Lead time, Mean lead time, Errors in Forecasting, Deviation of lead time etc. are considered in this safety stock calculation. Following this model, not only two factors (as we used demand and lead time), more than two factors could be used to determine safety stock. The best thing

of this model is none of the data used are from human judgment, all the data are derived from different calculations we had before head.

There could be limitations in this model, just like every other model has. But the limitation could be corrected when applied in the real working scenario. This way this model will more purified and applicable to all users of safety stock.

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